Design of Temperature Frequency Drift Compensation for Hydrogen Maser

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***Summary*—This paper presents a method to compensate for frequency drift in hydrogen maser caused by temperature fluctuations. The proposed solution involves constructing a model of the frequency-temperature coefficient using polynomial fitting. A testing environment comprising temperature measurement units, frequency measurement units, and frequency adjustment units was established to collect and analyze data. By fitting frequency data with temperature data using polynomial functions, the frequency drift can be accurately compensated in real-time. Experimental validation using Matlab demonstrated the feasibility and effectiveness of this approach, highlighting its potential for improving the frequency stability of hydrogen maser.**

Keywords—hydrogen maser, frequency stability, temperature compensation, polynomial fitting, Matlab simulation

I. INTRODUCTION

Due to the high stability and accuracy of quantum time frequency standards, as well as the fact that modern electronic technology can provide highly precise and reliable accumulation counting methods, the accuracy and precision of time frequency measurements are very high. In modern science and technology, the application of time and frequency measurement is very extensive[1], and almost all departments will use it. This greatly expands the application range of quantum frequency standards. The main performance of quantum frequency standards is the stability and accuracy of their standard frequencies.

At present, under ideal conditions, the maser will self-oscillate at a highly stable frequency. In practical use, there are various environmental factors that affect the state and frequency stability of self-oscillation. The temperature environment inside the hydrogen maser is one of the main factors limiting the frequency stability of the hydrogen maser[2]. In order to reduce the impact of changes in cavity resonant frequency on the frequency stability of hydrogen maser, it is necessary to compensate for the frequency drift value caused by temperature changes.

II. METHODS/RESULTS

This project aims to compensate for drift by constructing testing environments such as temperature measurement units, frequency measurement units, and frequency adjustment units. Firstly, the temperature measurement unit and frequency measurement unit are used to measure the data and establish a temperature characteristic (temperature coefficient) model of the atomic frequency standard. Subsequently, the temperature drift caused by temperature changes (compensation value) is calculated using this model, and the compensation value is quasi real-time feedback to the frequency adjustment unit. The original frequency signal is processed by the frequency adjustment unit and the output time signal is the time signal of the atomic frequency standard after temperature drift compensation (figure 1).

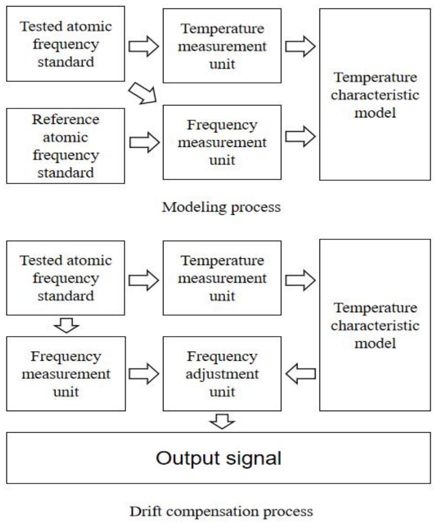

figure1. system structure diagram

1.Model construction phase

To accurately deduct the frequency drift of the hydrogen maser caused by temperature, we first measure and construct the frequency temperature coefficient curve of the hydrogen maser resonant cavity.

1.1 Analysis of frequency temperature effect in hydrogen maser resonant cavity

The core component of a hydrogen maser with a resonant cavity, and the frequency change of the resonant cavity is the main factor directly causing the output frequency of the hydrogen maser. The relationship between the change in output frequency Δf_0 of a hydrogen maser and the change in resonant cavity frequency Δf_c is given by cavity traction equation (1) [3].

$$\Delta f_0 = \Delta f_c \frac{Q_c}{Q_1} \tag{1}$$

Among them, Q_c is the Q value of the loaded cavity, and Q_1 is the effective atomic spectral line value. As can be seen from the above equation, a decrease in Δf_c can directly reduce the frequency change Δf_0. The frequency of the resonant cavity is influenced by changes in physical processes, with temperature being the main influencing factor. According to the above equation, as long as we can obtain the relationship between temperature and f_c, the frequency temperature coefficient curve, we can calculate the frequency drift of the hydrogen maser output caused by temperature changes through this curve.

1.2 Frequency temperature coefficient modeling

Firstly, determine the approximate range of temperature changes in the resonant cavity. Then, based on the approximate range of temperature changes, design highly sensitive temperature measuring elements within this temperature range. Then install both the temperature measurement element and the frequency measurement element onto the resonant cavity of the hydrogen maser, and let the hydrogen maser operate normally. At the same time, record the temperature data of the resonant cavity during operation and the corresponding frequency data of the resonant cavity to the computer. Run for a sufficient amount of time to ensure that as much frequency data at each temperature as possible is fully collected within the temperature fluctuation range during resonant cavity operation, and as much frequency data at each temperature as possible.

After precise measurement, model using the collected temperature and frequency coefficients. Among them, there are many corresponding frequency data for each temperature, and the average of these data is taken to minimize errors as much as possible. Then perform polynomial fitting on these data. Considering that the relationship between temperature and frequency may not be a simple linear relationship, we consider using polynomial fitting to model the data. The polynomial fitting model is an n-degree polynomial, which takes the form of:

$$y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \tag{2}$$

Where y is the response variable (dependent variable), x is the predictor variable (independent variable), a_n, a_{n-1}, $\cdots$, a_1, a_0 are coefficients of polynomials. The goal of polynomial fitting is to find these coefficients of the polynomial, so that the polynomial function P(x) can better fit a given m data points (x_i, y_i), minimizing the fitting error. There are many methods that can be used to minimize errors, and the most commonly used is the least squares method, which minimizes the sum of squared residuals (RSS):

$$\mathrm{RSS} = \sum_{i=1}^{m} (y_i - P(x_i))^2 \tag{3}$$

Here is the actual response value, and $P(x_i)$ is the response value predicted by the polynomial model. The process of minimizing RSS using the least squares method is as follows: First, construct the design matrix **X** containing the powers of the independent variable x, and the form is as follows:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^n \\ 1 & x_2 & x_2^2 & \cdots & x_2^n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_m & x_m^2 & \cdots & x_m^n \end{bmatrix} \tag{4}$$

Then construct a response vector **y** containing all the y_i values:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \tag{5}$$

Then use the least squares formula to calculate the coefficient vector **a**:

$$\mathbf{a} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{y} \tag{6}$$

The coefficient vector an obtained by solving contains coefficients fitted by polynomials. By substituting these coefficients into the polynomial model, the fitted polynomial function P(x) is obtained. At first, it is difficult to determine the degree of the polynomial here. Therefore, we can use polynomials of different degrees to fit and test residuals to select the most suitable degree for fitting.

Due to the fact that the hydrogen maser is connected to a temperature control system to control the temperature of the resonant cavity during actual operation, the temperature of the resonant cavity of the hydrogen maser will always be maintained within a certain range during operation, and we will fully measure the data within this range during measurement. Therefore, the fitted polynomial function does not need to predict the frequency at other temperatures, and there will be no overfitting problem. The P(x) here represents the frequency temperature coefficient to be used for the subsequent drift.

When P(x) is relatively complex, after constructing the coefficient curve, the data for each temperature can be first generated based on the accuracy of the temperature measuring element, and then directly queried during use. This way, the calculation of the correction amount will not be too slow due to overly complex frequency coefficients, ensuring the real-time performance of frequency compensation.

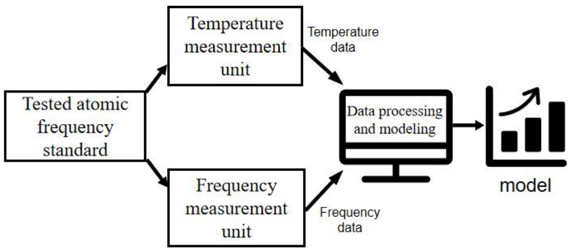

Figure 2. Modeling process of frequency temperature coefficient

2. Frequency compensation stage

After modeling through the above process, the polynomial function can be used for drift subtraction. During the actual operation of the hydrogen maser, the temperature t of the resonant cavity is recorded in real time, and then the temperature data t is transmitted to the computer. Here, t is the x in the polynomial function P(x). Whenever the temperature t of the resonant cavity changes, the resonant cavity frequency change Δf_c caused by the temperature change Δt can be calculated based on P(x). Subsequently, the drift Δf_0 of the output frequency of the hydrogen maser can be calculated based on cavity traction equation (1).

Then, the data of the drift amount xxx is input into a direct digital synthesizer (DDS) to synthesize a frequency signal of Δf_0. The frequency signal is input into a mixer together with the actual output frequency signal f_{real} of the hydrogen maser for combination, and finally outputs a frequency signal after deducting the temperature frequency drift, as shown in Figure 3 below:

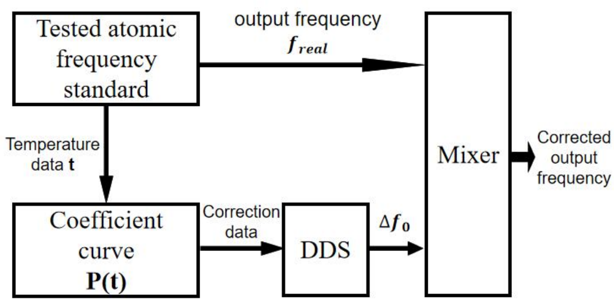

Figure 3. Process of frequency compensation

3. Experimental verification

To demonstrate the feasibility of using this method for drift deduction, experimental verification was conducted using Matlab. Firstly, generate temperature data that varies over time. Assuming that the relationship between the frequency and temperature of the resonant cavity satisfies a quadratic polynomial: f = a+bT+cT^2 (in reality, the relationship between frequency and temperature may be more complex than the assumed relationship). If a is taken as 1.0, b is taken as 0.01, and c is taken as 0.001, then the frequency data of the resonant cavity will be generated based on this. Random noise is introduced into the frequency data generated at each temperature to simulate possible errors. At the same time, in order to offset any errors that may occur, multiple data points will be taken for each temperature and the average will be taken.

Next, we use the generated resonant cavity frequency data and temperature data to fit and obtain the frequency temperature coefficient curve. Here, tests were conducted on the fitting effects of polynomials ranging from 1st to 5th degree, and the fitting effect of 2nd degree polynomials was the best. The fitted curve is as follows:

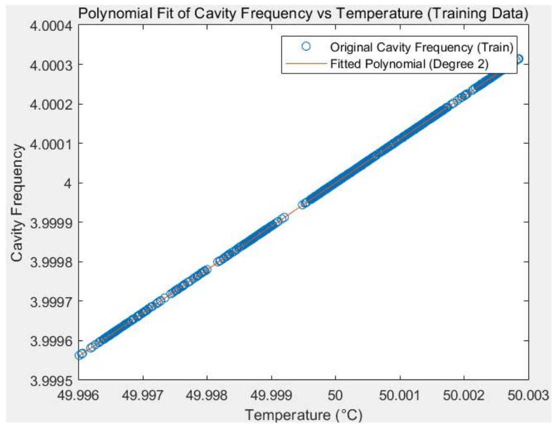

Figure 4. Frequency Temperature Coefficient Curve

The modeling process for frequency temperature coefficient ends here. Next, perform frequency compensation. Firstly, randomly generate a set of temperature data that varies over time, and use a quadratic polynomial: f=a+bT+cT^2 to generate frequency data that varies over time, representing the actual temperature during use. Here, 50 ℃ is taken as the optimal working temperature (the optimal working temperature can be selected according to the situation in the experiment), and then the temperature of the resonant cavity at each moment is recorded. The actual output frequency correction is calculated based on the difference between the resonant cavity temperature and the optimal working temperature (Figure 5). Then use this correction to compensate for the output frequency. The effect after compensation is shown in Figure 6, indicating that the output frequency has been well adjusted to the optimal operating temperature frequency after correction, and the expected correction effect is very good.

Here, the correction process is carried out after the generation of temperature and frequency data. In actual use, the temperature measurement and frequency correction process should be synchronized to ensure the effectiveness of frequency drift deduction.

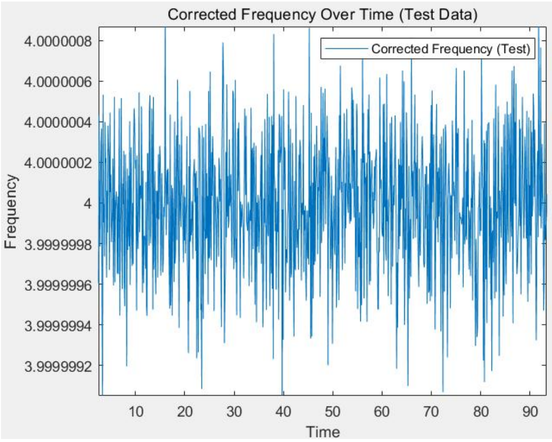

Figure 5. correction amount

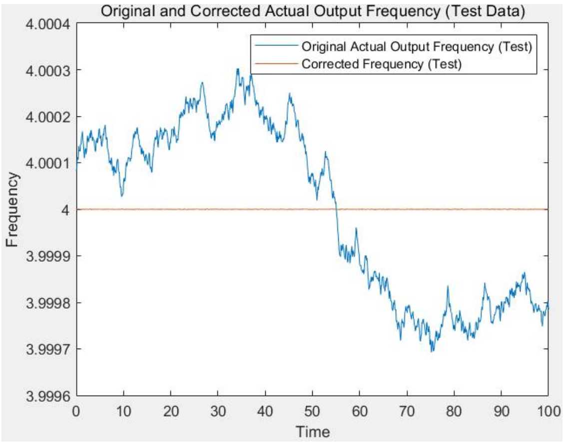

Figure 6. Correction effect

III. DISCUSSION/INTERPRETATION

The experimental results show that this design can effectively deduct frequency drift caused by temperature fluctuations. In practical use, the appropriate optimal temperature can be selected as the reference temperature for drift deduction based on theoretical deduction and other methods.

In hydrogen maser, the resonant cavity itself uses a temperature control system for temperature control, in order to reduce drift caused by temperature changes. However, since temperature cannot change suddenly, the adjustment will have a certain delay, which introduces a certain frequency drift in this process. By using this drift deduction method, this part of the frequency drift can be compensated in real time, improving the frequency stability of the hydrogen maser. Therefore, this design has practical significance.

IV. CONCLUSIONS

In conclusion, the method for temperature-induced frequency drift compensation in hydrogen maser presented in this paper has proven effective through experimental validation. The use of polynomial fitting to model the frequency-temperature relationship allows for accurate real-time compensation of frequency drift. This approach not only mitigates the impact of temperature fluctuations but also enhances the overall stability of the hydrogen maser. The experimental results indicate that a second-degree polynomial provides the best fit for the frequency-temperature coefficient curve, ensuring precise drift compensation. This method offers a practical solution for improving the performance and reliability of hydrogen maser in various applications.

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